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INSTABILITY OF A VIBRATIONAL FLUIDIZED BED

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The causes of fine- and large-scale instabilities of a vibrational fluidized bed are discussed and simple models are constructed; the reasoning presented is confirmed experimentally.

With vertical vibration of finely dispersed loads the free surface of the vibrationally fluidized bed very often proves to be nonhorizontal and the distribution of material over a cross section of the apparatus is nonuniform. This has been noted repeatedly in reports on vibrational fluidization, starting with the earliest ones (see [1-5], for example), but clearly insufficient attention has been paid up to now to the analysis of the causes of the nonuniformity phenomenon. At the same time, with the transition to large-scale installations this nonuniformity grows, and under certain conditions part of the vibrating bottom can prove to be entirely uncovered, which leads to instability in the operation of the installation and sometimes to its getting out of order prematurely.

Earlier the nonhorizontal nature of the free surface was connected with the vibrations not being vertical and with the amplitudes of vibrations of individual parts of the bottom being unequal [1, 2]. Special tests showed, however, that only loads of sufficiently large particles react to a change in the direction of the vibrational axis, but even for them the motion of the material is always directed toward the inclination of the axis in the lower part of the bed and in the opposite direction in the upper part, and it cannot be enlisted for an explanation of the observed nonuniformity. Loads of fine particles (∿0.1 mm in diameter) do not react at all to a small inclination of the vibrational axis, which agrees with the well-known data of [6] on vibrational transport and vibrational bunkering.

It seems obvious that the bias of the free surface and the nonuniformity of vibrational fluidization are due to the ordinary "hydrodynamic" instability of the "average" (unperturbed) state of a vibrational fluidized bed, formed under the action of complicated fields of gas-dynamic and viscoelastic forces arising in the process of the separation of the bed from and its falling onto the bottom as a result of the combined action of the relative motion of the gas and of waves of elastoplastic deformation propagating through the material of the load. The initial instability relative to small perturbations in the shape of the free surface leads to the formation of relatively slow secondary gas flows and to motion of the particles entrained by them, and this untimately causes the appearance of the observed nonuniformity. The decisive role of gas flows in the development of large-scale instability of a vibrationally fluidized bed is confirmed by the tests in [7], according to which the evacuation of the apparatus, the replacement of the closed bottom by a permeable porous one (which promotes a decrease in the swelling of the bed), and a transition to larger particles (for which the specific force of interaction with the gas is smaller than for fine ones) lead to leveling of the free surface and the disappearance of the nonuniformity.

Investigation of the stability of the average state of a bed is hindered by the fact that a theory which would permit an analytical description of the characteristics of this state

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itself in the general case has been absent until now. Relatively low beds at low relative vibrational accelerations, for which it proved possible to generalize Kroll's reasoning [1] and to construct a theory allowing for the expansion of the bed in the flight phase [8, 9], are the only exception. In the general case, therefore, it is necessary to resort to physical modeling of the system, using some experimentally established characteristics to describe its average state.

It is natural to assume that a granular material allows a relative deformation only in the flight phase, during which it can be modeled in a first approximation as some homogeneous continuous medium, permeable to gas, and with an effective density d, a viscosity μ, and a coefficient of hydraulic resistance to gas flow α. The motion of this medium, leading to the development of nonuniformities, is caused primarily by its interaction with secondary gas flows, whose appearance is due to perturbations in the gas pressure field. This field thus proves to be a basic characteristic of the state of a vibrationally fluidized bed. In the unperturbed state the pressure depends on the time t and the vertical coordinate z, as well as on the height h of the bed, the vibration frequency f, and the physical properties of the materials used, as on parameters. The latter quantity, its amplitude values, and the functions obtained through its time averaging have been measured and discussed repeatedly in the literature (see [5, 10], for example). As follows from the foregoing, the function p(z) obtained by averaging the oscillating pressure only over the part of the vibrational cycle corresponding to the flight phase, when the granular material can be entrained by gas flows, is of interest. This function has been determined experimentally for beds of corundum particles in tests discussed in detail below. A concept of them, as of the pressure distribution in the phases of flight and contact in general, can be obtained from Fig. 1, in which, besides a phase diagram for one such test, we present the corresponding dependences on h of the quantities

$$\gamma = -p(0), \quad \beta = -dp/dz|_{z=h}, \tag{1}$$

describing the average over the flight phase of the rarefaction below the bed and the average gradient of the rarefaction in the zone near the surface. For relatively shallow beds these quantities vary in correlation with h, which indirectly indicates the sign-constancy of the function p(z); for deeper beds the quantity p becomes an alternating-sign function of z and possesses extrema. The function p(z) can presently be obtained analytically only for very shallow beds, discussed in [8, 9].

We note that the diagram and the curves in Fig. 1 reflect the quasiperiodic ("beaded") character of the variation in the structure of a vibrationally fluidized bed with an increase in its height, noted earlier in [5]. This fact, which is of considerable independent interest, is also confirmed by phase diagrams in the h, f plane, an example of which is presented in Fig. 2. There is good agreement between the locations of the singular points on curves of the type shown in Fig. 1b and the variations in the structure of a bed at a given frequency, about which one can judge by examining intersections of the diagram of Fig. 2 with lines of f = const.

The characteristic time scale of the secondary flows is far longer than the vibration period, and therefore it is natural to perform averaging over a time interval which is considerably longer than this period but less than this time scale. The pulsations of parameters of the bed with the vibration frequency thereby prove to be smoothed out, and in the unperturbed state of the bed both the gas and the particles can be treated as stationary on the average.

First let us consider the evolution of small random perturbations in the shape of the free surface of the bed, $z = h + \delta$, $\delta \ll h$, in the x, z plane. These perturbations are accompanied by the appearance of pressure disturbances $p'(t, x, z)$ superimposed on the average pressure field p(z) and of secondary motion of the gas and the granular material with velocities $\mathbf{v}(t, x, z)$ and $\mathbf{w}(t, x, z)$, with the inequality $v \gg w$ being valid because of the large difference between the densities of the gas and the particle material. For the same reason we can neglect inertial forces in the gas in comparison with the forces of hydrodynamic interaction with the particles and, taking these forces as linear with respect to the relative velocity $\mathbf{v} - \mathbf{w} \approx \mathbf{v}$, we can describe the gas motion with the help of the ordinary Darcy equation. Also using the equation of conservation of mass of an incompressible gas, we have

$$\mathbf{v} = -\alpha^{-1}\nabla p', \quad \nabla p' = 0. \tag{2}$$

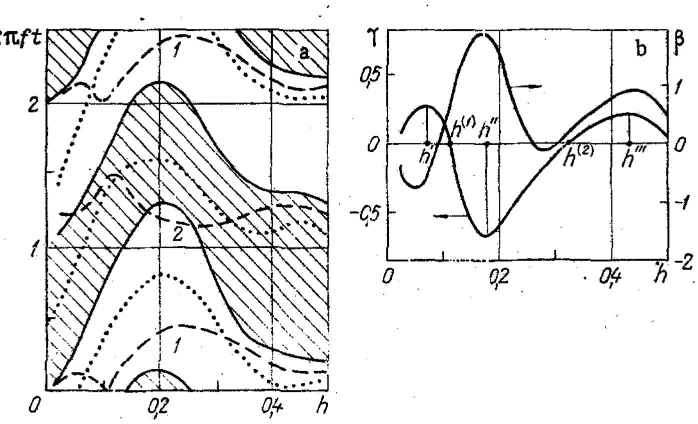

Fig. 1. Phase diagram (a) and dependences of the parameters γ and β on h (b) for a bed of 0.08 mm corundum particles with a vibration amplitude of 2.7 mm and a frequency of 20 Hz. Solid lines: limits of parts of the vibrational cycles corresponding to excess pressure and to rarefaction below the bed (the rarefaction region is hatched); dashed lines (1 and 2): phase angles of separation from and falling of the bed onto the bottom, respectively; dotted lines: phase angles corresponding to maximum (amplitude) values of pressure and rarefaction. γ, kPa; β, kPa/m; h, m.

In the formation of the boundary conditions we must use the equality of the total gas pressure above the bed to the atmospheric pressure (which it is convenient to take as the zero pressure reading) and the impermeability of the bottom to the gas (which corresponds to a closed bottom). Thus, with allowance for the smallness of $\delta = \delta(t, x)$, we obtain

$$p'|_{z=h} - \beta\delta = 0, \quad \partial p'/\partial z|_{z=0} = 0. \tag{3}$$

The motion of the granular material, treated in the approximation of a homogeneous, Newtonian, continuous medium on which an external volumetric force $\alpha(\mathbf{v} - \mathbf{w}) \approx \alpha\mathbf{v}$ acts, is controlled by the ordinary Navier–Stokes equations

$$\frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z} = 0, \quad d\,\frac{\partial \mathbf{w}}{\partial t} = -\nabla\pi + \mu\Delta\mathbf{w} + \alpha\mathbf{v}, \tag{4}$$

where π is the effective pressure in this fictitious continuous medium, characterizing the interaction between the moving particles. Introducing the stream function φ in the standard way, from (4) we obtain the equation

$$\frac{\partial}{\partial t}\,\Delta\varphi = \nu\Delta^2\varphi + \frac{\alpha}{d}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) = \nu\Delta^2\varphi, \tag{5}$$

with the latter equality in (5) following from the potential nature of the filtrational motion of the gas. For π from (4) we obtain the equation

$$-\frac{\partial \pi}{\partial z} = -d\,\frac{\partial^2\varphi}{\partial t\partial x} + \mu\,\frac{\partial}{\partial x}\,\Delta\varphi + \frac{\partial p}{\partial z}\,. \tag{6}$$

In the formation of the corresponding boundary conditions we need to use the impermeability of the bottom to the particles, the reduction of the normal and tangential stresses to zero at the free surface of the bed, and the reduction of the tangential stresses to zero at the lower surface of the bed (quite understandable physically if one remembers that we are only considering the flight phase of the bed and one considers that the normal stress at this surface is different from zero, generally speaking, as a result of the impact interactions of the granular material with the bottom). In addition, there is a kinematic condition connecting the vertical component of the particle velocity at the free surface with the rate of change of the coordinate of this surface. Again, using the model of a homogeneous Newtonian medium, we obtain

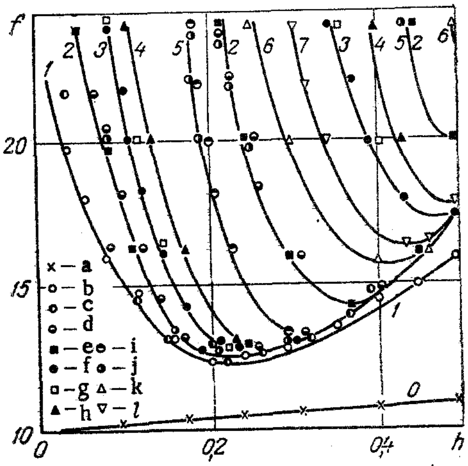

Fig. 2. Summary diagram of states of a vibrationally fluidized bed of 0.12 mm corundum particles; vibration amplitude 2.7 mm: 0) line of start of tossing (a); 1) start of appreciable expansion (b) and fluidization (c); 2) start of swelling, appearance of bubbles (d), and of maximum value of phase angle of separation (e); 3) maximum swelling (f) and minimum values of phase angle of separation (g); 4) maximum amplitude of pressure below the bed and maximum fountaining (h); 5) minimum swelling (i) and disappearance of bubbles (j); 6) minimum amplitude of pressure below the bed (k); 7) development of irregular tossing mode (l). f, Hz.

$$w_z|_{z=0} = 0, \quad \partial w_x/\partial z|_{z=h} = 0, \quad (-\pi + \mu \partial w_z/\partial z)_{z=h} = 0, \quad \partial w_x/\partial z|_{z=0} = 0, \quad w_z|_{z=h} = \partial\delta/\partial t. \tag{7}$$

As usual, in an investigation of stability in the small it is sufficient to consider only the simplest wave of the perturbations, i.e., to take

$$\{p,\ \mathbf{v},\ \mathbf{w},\ \pi,\ \varphi,\ \delta\} = \{P(z),\ \mathbf{V}(z),\ \mathbf{W}(z),\ \Pi(z),\ \Phi(z),\ \Delta\}\, e^{i(\omega t + kx)}. \tag{8}$$

The equations and boundary conditions for the amplitudes in (8) are easily obtained from (2), (3), (5)-(7), and the definition of the stream function. In particular, we have

$$P = \beta\Delta \frac{\operatorname{ch} kz}{\operatorname{ch} kh}, \quad V_x = -ik \frac{\beta\Delta}{\alpha} \frac{\operatorname{ch} kz}{\operatorname{ch} kh}, \quad V_z = -k \frac{\beta\Delta}{\alpha} \frac{\operatorname{sh} kz}{\operatorname{ch} kh}. \tag{9}$$

The general solution of the equation for Φ obtained from (5) has the form

$$\Phi = \Phi_1 e^{kz} + \Phi_2 e^{-kz} + \Phi_3 e^{\lambda z} + \Phi_4 e^{-\lambda z}, \quad \lambda^2 = k^2 + i\omega/\nu, \tag{10}$$

it being easy to obtain a system of five homogeneous, linear, algebraic equations for the constants in (10) and for the quantity Δ from (6)-(9). The condition for the presence of a nontrivial solution of this system leads to the secular equation

$$k^3 \operatorname{cth} \lambda h - \lambda^3 \operatorname{cth} kh = \frac{\beta}{d\nu^2} \frac{k}{\lambda}, \tag{11}$$

which determines the frequency ω of the perturbations in the form of a function of the wave number k and the parameters of the vibrational fluidized bed. States of the bed for which the imaginary part of ω is negative will be unstable. The value of k yielding the maximum of this quantity characterizes the wavelength of the most rapidly growing perturbation.

It is not hard to investigate Eq. (11) numerically in the general case, but to obtain the main qualitative results it is sufficient to consider the very probable case when $|\omega| \ll \nu k^2$. With the accuracy of terms of first order with respect to $|\omega|/\nu k^2$ we obtain from (11)

$$\omega = 2i \frac{\beta h}{d\nu} \frac{1}{kh(3\operatorname{cth} kh + kh \operatorname{sh}^{-2} kh)}. \tag{12}$$

Thus, the initial perturbations evolve monotonically with time without oscillations; they grow in the case when $\beta < 0$, i.e., when the rarefaction intensifies with greater distance into the bed from the free surface, and they die out in the opposite case. When $\beta < 0$ the pressure at points below "depressions" in the free surface is higher than the pressure at points lying in the same horizontal plane but below "humps." As a result, circulating gas flows develop in which gas penetrates into the bed through depressions and leaves it through humps.* Particles are entrained by these flows in the same direction, and this leads to deepening of the depressions and growth of the humps. If $\beta > 0$ then the flows are oppositely directed to those discussed and lead to the filling in of depressions and the smoothing out of initial humps.

An instability of the investigated type obviously is small-scale, involves only the surface section of a vibrational fluidized bed, and leads to the appearance of ripples at its surface, observed in [3], for example; the conditions for the realization of this instability are easily determined by examining the form of the p(z) curve and its dependence on the parameters of the bed. Thus, under the conditions of the experiment illustrated in Fig. 1 ripples appear in shallow beds ($h \leqslant 10$ cm), disappear with an increase in the height of the bed, and appear again at $h \simeq 25$-30 cm.

With the growth of the initial perturbations (or upon the development for some reason of perturbations which are not small from the very start) the linear theory presented ceases to be applicable. Physically this means, in particular, that not only the state of the surface zone of the bed but also the perturbed pressure distribution in its interior, which depends strongly both on the character of the accumulated perturbations and also on the detailed form of the entire unperturbed distribution p(z), starts to play a role in the generation of the horizontal gas flows stimulating the development of secondary circulating flows. Although it is scarcely possible to construct a general theory for large-scale perturbations, a concept of the physical causes and the general trend of their evolution can be obtained from an examination of the simplest examples.

As an example, let us consider the experiment shown in Fig. 3a. A vibrating load with an initial height h_0 is divided by a vertical barrier into two equal piles so that there is a narrow gap between its lower boundary and the vibrating bottom. Since the width of the gap is far less than h_0 (but greater than the vibration amplitude, of course), the distributions p(z) in the two sections of the apparatus are established almost independently, with the interaction between the piles being determined only by the amounts of rarefaction under them (i.e., by the respective values of γ). If $h_0 < h'$ then $d\gamma/dh > 0$ (see Fig. 1b) and a perturbation δh in the heights of the piles ($h_1 = h_0 - \delta h$, $h_2 = h_0 + \delta h$) leads to the fact that the rarefaction under the higher pile (on the right in Fig. 3a) is greater than the rarefaction under the lower pile. As a result, a gas flow through the gap develops which is directed from the first section to the second during the flight phase. The average particle flow is also directional. Ultimately in the case of $2h_0 < h^{(1)}$, where $h^{(1)}$ is the second root of the equation $\gamma(h) = 0$, following the root $h = 0$, all the particles will be transferred into the second section, as shown in the first part of Fig. 3a. If $2h_0 > h^{(1)}$ then an equilibrium state is evidently established such that $h_2 > h_1 > 0$ and $\gamma(h_1) = \gamma(h_2)$. In any case, the complete or partial vibrational bunkering of the finely dispersed granular material will thus occur.†

Now let h_0 satisfy the inequalities $h' < h_0 < h''$, so that $d\gamma/dh < 0$. It is easy to see that in this case a perturbation δh in the heights of the piles causes the development of a difference in the rarefactions under them, which stimulates the reverse transfer of particles, equalizing the piles. In this height interval the state $h_1 = h_2 = h_0$ is stable (see second part of Fig. 3a). In the interval of $h'' < h_0 < h'''$ the state with piles of equal height again proves to be unstable: the directional transfer of granular material proceeds until the establishment of a final stable state for which $\gamma(h_1) = \gamma(h_2)$, as shown in the third part of Fig. 3a. The reasoning presented is easily extended to loads with higher values of h_0 also; on the whole, states corresponding to the ascending branches of the curve $\gamma(h)$ are unstable while those corresponding to the descending branches are stable. It is also easy to analyze situations when the initial heights of the piles are different: depending on the loca-

*We note that the development of a nonuniform field p(z) in a vibrational fluidized bed which is in a stationary state on the average represents an internal property of the bed; this field obviously does not lead to the development of directed vertical movement of the gas.

†Actually, a thin "shelf" of material covering the gap always remains in the empty section. We also note that the presence of a slow transfer of particles from one section to the other hardly affects the establishment of the unperturbed pressure distributions p(z) in the two of them.

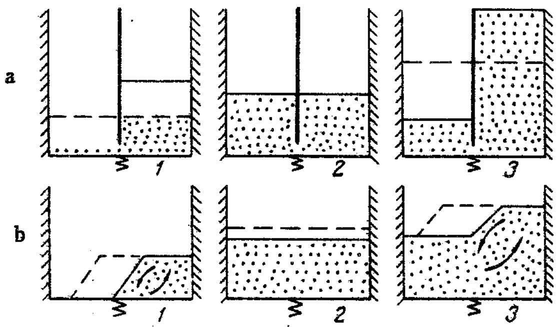

Fig. 3

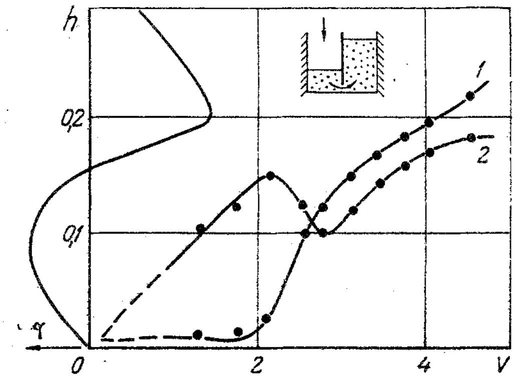

Fig. 4

Fig. 3. Sketches (1, 2, 3) of states of a vibrational fluidized bed with an increase in the volume of the load: a) bed divided into two sections by a barrier; dashed lines: initial state; b) free bed; dashed lines: change in bed configuration upon the addition of a small portion of material.

Fig. 4. Experimental dependences of heights of piles in neighboring sections of a vibrationally fluidized bed of 0.12 mm corundum particles with a vibrational amplitude of 2.7 mm and a frequency of 15 Hz on the dimensionless total volume of the bed. The corresponding dependence γ(h) is shown arbitrarily on the left.

tion of the corresponding points on the curve of γ(h), both the further vibrational bunkering with the establishment of a state in which the piles differ even more strongly in height and the complete or partial recuperation are possible. In principle, by analyzing the dependence of γ on other parameters it is easy to investigate the conditions of the onset of vibrational bunkering and recuperation as these parameters vary.

The above reasoning is fully confirmed by tests set up on loads of fine corundum particles of different diameters with different vibration frequencies and initial bed heights. The results of one such test are presented in Fig. 4 as an example: the dependences of h_1 and h_2 on the arbitrary variable V, representing the dimensionless total volume of the load, are presented. The corresponding curve of γ(h) is also shown arbitrarily in Fig. 4. The volume of the load was varied under the test conditions by adding material to the first section. The almost complete vibrational bunkering of the material is observed for shallow beds. With an increase in the volume of the load the levels of the piles are equalized, and this occurs just at those values of h which, with the accuracy of the experimental error, coincide with the value h' producing the first extremum of the function γ(h). Then the heights of the two piles grow equally; this stable state just corresponds to the growing branch of γ(h). (The small "underrecuperation" is evidently explained by the influence of wall frictional forces.) Finally, vibrational bunkering begins again in the region of the second extremum of γ(h).

A variant of the described tests in which one of the sections is throttled or an excess pressure or rarefaction is artificially created in it is interesting. Tests of such a type with the submersion of a special "bucket" or pipe in a vibrationally fluidized bed are described in [11, 12]. In this case the pressure above the pile in the indicated section differs from atmospheric pressure and affects the absolute value of the pressure under the pile also, which shows up in the transfer of gas and granular material, of course. In contrast to the situation discussed above, when the evolution of the system was determined only by the gas pressure in the bed during the flight phase, here the pressure during the contact phase is also important. Actually, although the granular material is stationary during contact, the gas flow continues and affects the pressure established in the throttled section.

Now let us discuss the phenomena occurring in the case when the barrier is raised, i.e., the size of the gap is increased and even becomes comparable with the height of the bed. It is clear that the flows of gas and particles are distributed over the entire "free" boundary between the piles in the neighboring sections and the interaction between them is determined not only by the values of the pressures at $z = 0$ but generally by the distributions p(z) as a whole. In this case one can no longer assume that these distributions are independent and coincide with the distributions for isolated vibrationally fluidized beds of the corresponding heights. The limiting case is obviously reached when the barrier is removed (Fig. 3b).

If the bed is shallow enough that the distribution p(z) is close to monotonic (see [8-10]), then the relationship between the pressures in the neighboring sections at different levels can be judged approximately from the relationship between the corresponding values of γ. In this case all the reasoning presented above remains valid in a qualitative respect. In particular, if the average bed height in Fig. 3b is less than a value close to h' then any defect in the free surface will be intensified, with the ultimate establishment of a stable state, shown in the first part of Fig. 3b: part of the apparatus is occupied by a flat bed of height ∿h' while part of the apparatus is not loaded; the shape of the transitional zone is determined by the value of the dynamic angle of repose. Upon the addition of material a uniform distribution of granular material over the vibrating bottom becomes stable (second part of Fig. 3b). For deeper beds such a distribution again ceases to be stable, and from the above standpoints one would exect the establishment of the state shown in the third part of Fig. 3b. In reality, however, in a deep bed the distribution p(z) is essentially monotonic and so it cannot be judged only from the quantity $\gamma = p(0)$. In this case not only the amounts but also the directions of the transfers of gas and particles vary over the height of the bed, with the development of complicated circulation loops which also affect the dynamic equilibrium. Therefore, the actual state of the bed will differ from the idealized state considered. We note that all the indicated states have been observed and discussed repeatedly [5].

NOTATION

d, density of medium modeling the granular material; f, vibrational frequency; h, bed height; k, wave number of perturbations; p, pressure; p', P, perturbation of the field of gas pressure and its amplitude; t, time; **v**, **V**, velocity of gas filtration and its amplitude; **w**, **W**, velocity of granular material and its amplitude; x, z, horizontal and vertical coordinates; α, coefficient of hydraulic resistance; β, γ, rarefaction gradient near free surface of the bed and rarefaction below the bed, averaged over the flight phase; δ, Δ, perturbation of shape of the free surface and its amplitude; λ, parameter introduced in (10); μ, viscosity of medium modeling the granular material; $\nu = \mu/d$; π, Π, effective pressure inside the granular material and its amplitude; φ, Φ, stream function of granular material and its amplitude; ω, complex frequency of perturbations.

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